Energy-efficient adaptive power allocation in orthogonal frequency division multiplexing-based amplify-and-forward relay link

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Abstract: Recently, improving energy efficiency has been a new tendency in wireless relay communication. However, the introduction of relay technology may increase the system capacity, but is often followed by a more severe energy-efficiency problem when compared with traditional single-hop system. In this study, based on orthogonal frequency division multiplexing-based frequency-selective channel of amplify-and-forward relay link, a suboptimal two-step power allocation algorithm of subcarriers is proposed to maximise the energy efficiency. The proposal includes two steps. First, a suboptimal subcarrier matching is achieved for introducing a virtual direct link, which may reduce the dimensionality of the power allocation. Second, the allocation problem of relay network is simplified as that in traditional cellular network. Thus, the optimal power allocation of virtual direct link can be obtained by one-dimensional search method. Furthermore, the effects of rate and power constraints on the energy-efficiency problem are analysed. Simulation shows that the suboptimal solution of the proposed algorithm is very close to the optimal solution, and the proposed energy efficient power allocation method can achieve the highest energy efficiency, compared with the rate adaptive and margin adaptive optimisations.

1 Introduction

In the next generation communication, the relay technology is introduced in 802.16j, 802.16 m and LTE-advanced to support broader signal coverage, higher data transmission rate and faster mobility. Nowadays, there are two typical relay modes, that is, amplify-and-forward (AF) relay and decode-and-forward (DF) relay. Currently, saving resources have been the new tendency of technology economic development. Thus, the objective of resource allocation in the wireless communication should tend towards improving the resources utilisation, especially the power utilisation. In other words, improving the transmit rate per watt defined as energy efficiency in [1] is becoming a vital problem for wireless communications. However, in the field of resource allocation in relay network, the research mainly focuses on the rate adaptation optimisation (RAO), the throughput maximisation [2–5] and the margin adaptive optimisation (MAO), the transmit power minimisation [6–7]. The AF relay mode is considered in [2–4], which maximises sum rate or spectral efficiency through the resource allocation in orthogonal frequency division multiple access (OFDMA) networks. Yin et al. [2] proposed a power allocation scheme for single relay link, whereas the authors [3, 4] studied the joint optimisation of subcarrier and power allocation based on multiuser relay system. The DF relay mode is studied in [5–7]. To maximise the total channel capacity, the subcarrier matching (SM) and power allocation were well investigated in [5]. To minimise the total transmit power, the joint subframe, subcarrier and bit allocation under rate constraints were proposed with a fixed set of discrete modulation levels in [6] and with continuous set of modulation in [7]. Nevertheless, both RAO and MAO are single objective optimisation so cannot reach the optimal energy efficiency. Energy-efficiency optimisation (EEO) can be seen as a multi-objective optimisation which minimises the power consumption while maximising achievable system throughput. MAO can save more energy and RAO can improve throughput, they both can promote energy efficiency. However, RAO and MAO are not equivalent to EEO and are the special cases of EEO under rigorous constraints. When the constraints are very loose, the data rate from MAO is very low and the high rate cannot be guaranteed. At the same time, the power consumption from RAO is very large. Therefore, the research of resource allocation mainly focuses on RAO and MAO, but not much work of the resource allocation could be found in EEO.

In term of the energy-efficiency problem, that using the output power as the performance metric only captures a small fraction of the overall power budget of wireless networks may lead to incomplete and potentially misleading conclusions [1]. In order to evaluate the accurate energy efficiency of a wireless network, the circuit power operating the entire access network should be taken into account. In [8–10], the energy-efficiency problem under the
result, based on AF relay system and frequency selective channel, there has not been any research result on the energy-efficient power allocation for each subcarrier which is worthy of further investigation.

In this paper, a novel adaptive power allocation is considered to maximise the energy efficiency in OFDM-based AF relay link. The power allocation of relay link is decomposed into two steps. In the first step, only fast fourier transformation (FFT) and inverse fast fourier transformation (IFFT), which can reorder the received subcarrier signals, are performed, so AF relay does not change the code rate, modulation scheme and the number of subcarriers. According to the characteristics of SNR in relay system, two-hop relay link is converted to a virtual direct link. Thus a SM algorithm over relay link is established to obtain the virtual direct link gain for decreasing the algorithm complexity of power allocation. In the second step, the low-complexity algorithm of [8] is used to allocate power of each subcarrier in virtual direct OFDM frequency-selective channel for maximum total energy efficiency. Furthermore, the cases with minimum rate and maximum power constraints are analysed.

The rest of this paper is organised as follows. In Section 2, the system model and the formulation of joint optimisation are presented. The SM method based on virtual direct link and the optimal energy-efficient power allocation algorithm are presented in Section 3. Section 4 analyses the case with constraints. A performance evaluation of the proposed algorithm is given in Section 5, and conclusions are drawn in Section 6.

2 System model and problem formulation

Based on an OFDM-based AF relay link, it is supposed that there is one source node (SN), one relay node (RN) and one destination node (DN) in the link. A typical application of this scenario is the vehicle-to-infrastructure communication between base station and driver’s mobile station through the relay station mounted on the roof of a train. There are K subcarriers in system, with the bandwidth of W Hz. The RN is located in a line between SN and DN, as shown in Fig. 1. The RN in this paper operates in AF and half-duplex mode. There are two slots in each transmission: odd slot and even slot. In the odd slot, the signal is transmitted in SN–RN link, which is called the first hop. In the even slot, the signal is transmitted in RN–DN link, which is called the second hop. Each hop is allocated the equal amount of subcarriers, that is, K subcarriers. It is assumed that the transmissions experience path loss, Rayleigh fading and shadowing. Rayleigh fading is assumed to be flat in each subcarrier. Therefore OFDM transmission in frequency-selective channel is considered, and the channel gain of each subcarrier is different. In this system, the SN serves as the central planner for the cell, controlling the power allocation in each hop with all channel state information (CSI).

Let $p_i^s$ denotes the transmit power of the $i$th subcarrier in the first hop, and $p_j^r$ denotes the transmit power of the $j$th subcarrier. 4

**Fig. 1** System model $55 \times 13$ mm
subcarrier in the second hop. Then the transmit power vectors on all subcarriers in the first and second hops can be denoted by

\[
P_1 = [p_1^1, p_2^1, \ldots, p_K^1]^T
\]

(1)

\[
P_2 = [p_1^2, p_2^2, \ldots, p_K^2]^T
\]

(2)

and \(P_1, P_2 \in \mathbb{R}_+^K\), \(\mathbb{R}_+^K\) denotes the set of non-negative real \(K\)-vectors.

Corresponding to subcarrier \(i\) in the first hop, the matched subcarrier in the second hop is assumed as \(j = c(i)\). According to the result of SM, power allocation and CSI of each hop, the signal-to-noise ratio (SNR) of each subcarrier pair \(i-j\) can be obtained. The noise power spectral density is denoted as \(N_0\). For the convenience of mathematical expression, the received SNR \(\epsilon_i\) of the \(i\)th subcarrier pair in the relay link is denoted as that in [17]

\[
\epsilon_i(p_1^i, p_2^i) = \frac{p_1^i g_i^1 \times p_2^i g_i^2}{p_1^i g_i^1 + p_2^i g_i^2}
\]

(3)

where

\[
g_i^1 = \frac{|H_i^1|^2}{N_0 W}, \quad g_i^2 = \frac{|H_i^2|^2}{N_0 W}
\]

denote the channel gains of subcarriers in each hop, respectively. \(|H_i^1|^2, |H_i^2|^2\) denote the power gains of subcarriers in each hop, respectively. The form of (3) has the advantage of mathematical tractability over that in theoretical SNR in addition to be a tight upper bound for the form of theoretical SNR, particularly at high average SNR. Then the rate of \(i\)th subcarrier pair in relay link can be denoted as

\[
r_i = \frac{W}{2} \log_2 (1 + \epsilon_i)
\]

(4)

The total rate of the user is

\[
R(P_1, P_2) = \sum_{i=1}^{K} r_i = \sum_{i=1}^{K} \frac{W}{2} \log_2 \left(1 + \frac{p_1^i g_i^1 \times p_2^i g_i^2}{p_1^i g_i^1 + p_2^i g_i^2}\right)
\]

(5)

During transmission, the devices transmitting signal incur additional circuit power which is relatively independent of the transmission rate [1]. The circuit power represents the average energy consumption of device electronics, and this portion of energy consumption excludes that of the RF (radio frequency) power amplifier. We denote the circuit power as \(P_c\), the total transmit power as \(P_T\), the power amplifier efficiency as \(\zeta\). Then, the overall power consumption is

\[
P_{total} = P_c + P_T = P_c + \frac{1}{\zeta} \sum_{i=1}^{K} (p_1^i + p_2^i)
\]

(6)

For energy efficient communication, the objective is to maximise the total throughput with a given amount of energy. Therefore, in a duration \(\Delta t\), the consumed energy is \(\Delta e = \Delta t \cdot P_{total}\). It is desirable to send the maximum rate by reasonable power allocation of each hop for maximising

\[
\frac{R(P_1, P_2) \Delta t}{\Delta e}
\]

(7)

which is equivalent to maximising

\[
U(P_1, P_2) = \frac{R(P_1, P_2)}{P_{total}} = \frac{R(P_1, P_2)}{P_c + P_T}
\]

(8)

where \(U(P_1, P_2)\) denotes the energy efficiency, the unit of which is bits/joule. The optimal energy-efficient power allocation achieves maximum energy efficiency, that is

\[
(P_1^*, P_2^*, c^*(i)) = \arg \max \limits_{P_1, P_2, c(i)} U(P_1, P_2) = \arg \max \limits_{P_1, P_2, c(i)} \frac{R(P_1, P_2)}{P_c + P_T}
\]

(9)

As seen from (9), the optimal energy efficiency is achieved by simultaneously allocating reasonable power vectors in two hops. Unfortunately, the energy-efficient power allocation is a combinatorial mixed integer programming problem. The problem is computationally extensive when the number of variables is large, that is, \(N \cdot P\) hard, and thus it is intractable. However, when \(c(i)\) is given, (9) is concave function, so the optimisation problem is a convex optimisation problem. The optimal power allocation can be achieved by the binary search algorithm. In order to simplify this problem, a suboptimal power allocation is proposed and solved by two steps. The SM is separated from the power allocation. If the solutions of SM and the power allocation are both optimal, the optimal solution of (9) is obtained.

### 3 Energy-efficient power allocation algorithm

Owing to the AF relay, the SNR of subcarrier pair \(i-j\) is determined by \(p_1^i, p_2^i\), which can be explained by (3). Therefore we firstly propose a SM method for converting two-hop relay link to a virtual direct link and simplifying the energy-efficiency problem of relay link to that of the traditional single-hop link. Then the single-hop link allocation algorithm is used to allocate power to maximise the energy efficiency.

By separately performing SM, a suboptimal power allocation algorithm is yielded which is considerably simpler to implement than the original problem. The suboptimal algorithm will be presented in following subsections.

#### 3.1 SM method

In this subsection, a SM method is proposed. Based on SM, the two-hop relay link can be converted to a single-hop link, if the channel gain between SN and DN is known. Hence, relay transmissions are transformed to corresponding direct transmissions that have virtual link with the same channel gain. The virtual direct transmission channel gain is determined to maximise the received SNR of a relay transmission. It can be achieved by optimal power distribution between source and relay. This approach enables all the relay transmission to be regarded as direct transmission according to the result of SM.
3.1.1 Virtual direct transmission channel gain: A virtual direct is assumed a transmission between SN and DN, corresponding to the relay transmission in association with SN and RN on subcarrier pair $i-j$. The SNR of virtual direct transmission on subcarrier pair $i-j$ equals to the one of relay link. If the amount of transmit power allocated to subcarrier pair $i-j$ is $\hat{p}_{ij}$, by using the similar idea of [18], the virtual direct transmission channel gain $\hat{g}_{ij}$ is derived by solving the following maximisation problem

$$\hat{p}_{ij}\hat{g}_{ij} = \max_{p_i, p_j} \left( p_i, p_j \right) = \max_{p_i, p_j} \frac{p_i^2 \hat{g}_{i}^2 \cdot p_j^2 \hat{g}_{j}^2}{p_i^2 \hat{g}_{i}^2 + p_j^2 \hat{g}_{j}^2} \quad (10)$$

s.t. $\hat{p}_{ij} = p_i + p_j \quad (10a)$

$$p_i, p_j \geq 0 \quad (10b)$$

From (10), it can be seen that a virtual direct link is constructed from SN to DN with the channel gain $\hat{g}_{ij}$ and equivalent to the optimal relay transmission for a given power $\hat{p}_{ij}$.

**Proposition 1:** The virtual direct transmission channel gain is obtained by the channel gains $\hat{g}_{i}, \hat{g}_{j}$ and it is independent of total power $\hat{p}_{ij}$ on subcarrier pair

$$\hat{g}_{ij} = \frac{d_{ij}}{d_{ij} + 1} \frac{\hat{g}_{i}\hat{g}_{j}}{\hat{g}_{i} + \hat{g}_{j}} \quad (11)$$

where $d_{ij} = \sqrt{\frac{\hat{g}_{i}}{\hat{g}_{j}}}$.  

**Proof:** The Hessian function of (10) is represented as

$$e_i''(p_i^1, p_j^1) = 2 \begin{bmatrix} (\hat{g}_{i})^2 (\hat{g}_{j})^2 (p_j^1)^2, (\hat{g}_{i})^2 (\hat{g}_{j})^2 (p_i^1)^2 \\
(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2, (p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2 \\
(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2, (p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2 \\
\end{bmatrix} - \frac{(\hat{g}_{i})^2 (\hat{g}_{j})^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2} \times \frac{(\hat{g}_{i})^2 (\hat{g}_{j})^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2}$$

We can easily obtain that $e_i''(p_i^1, p_j^1)$ is negative semi-definite matrix. Hence, (10) is concave and has a unique optimal solution. By using the method of Lagrangian multiplier, it is obtained that

$$L(p_i^1, p_j^1) = \frac{p_i^1 \hat{g}_{i} \times p_j^1 \hat{g}_{j}}{p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j}} - \lambda \left( p_i^1 + p_j^1 - \hat{p}_{ij} \right) \quad (12)$$

where $\lambda$ is the non-negative Lagrangian multiplier. Take the derivatives with respect to $p_i^1, p_j^1$, respectively, as follows

$$\frac{\partial L}{\partial p_i^1} = \frac{g_i^2 (p_i^1)^2 (\hat{g}_{j})^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2} - \lambda = 0 \quad (13)$$

$$\frac{\partial L}{\partial p_j^1} = \frac{(\hat{g}_{i})^2 (p_j^1)^2 \hat{g}_{j}^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2} - \lambda = 0 \quad (14)$$

From (13) and (14), it is obtained that

$$\frac{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2} = \frac{(\hat{g}_{i})^2 (p_j^1)^2 \hat{g}_{j}^2}{(p_i^1 \hat{g}_{i} + p_j^1 \hat{g}_{j})^2} \quad (15)$$

Simplifying (15), it is obtained that

$$p_i^1 = \sqrt{\frac{\hat{g}_{i}}{\hat{g}_{j}}} p_j^1 \quad (16)$$

Substituting (16) and (10.a) into (10), (11) can be obtained. □

3.1.2 SM for virtual channel gain: Let $G_i = [g_{i1}, g_{i2}, \ldots, g_{iN}]$ and $G_j = [g_{j1}, g_{j2}, \ldots, g_{jN}]$ denote the channel gain sets of subcarriers over the first and second hops, respectively, then sort the elements in $G_i$ and $G_j$, respectively, in ascending order. Two new channel gain sets of subcarriers over the two hops are achieved, which are $G_{1i} = [g_{1i}, g_{2i}, \ldots, g_{Ni}]$ and $G_{1j} = [g_{1j}, g_{2j}, \ldots, g_{Nj}]$, where $g_{1i} \leq g_{2i} \leq \cdots \leq g_{Ni}$ and $g_{1j} \leq g_{2j} \leq \cdots \leq g_{Nj}$, $n$ is an arbitrary positive integer. By simplifying (11), we can obtain

$$\frac{1}{\sqrt{\hat{g}_{i}}} = \frac{1}{\sqrt{\hat{g}_{i1}}} + \frac{1}{\sqrt{\hat{g}_{j}}} \quad (17)$$

First, we consider the case that each hop only has two subcarriers. Here, there are two ways to match the subcarriers: (i) Subcarrier 1 of the first hop is matched to Subcarrier 1 of the second hop, and Subcarrier 2 of the first hop is matched to Subcarrier 2 of the second hop, that is, $g_{1i} \sim g_{1j}$ and $g_{2i} \sim g_{2j}$; (ii) Subcarrier 1 of the first hop is matched to Subcarrier 2 of the second hop, and Subcarrier 2 of the first hop is matched to Subcarrier 1 of the second hop, that is, $g_{1i} \sim g_{2j}$ and $g_{2i} \sim g_{1j}$. For the two ways of matching subcarrier, the relationship between the virtual direct link gains can be expressed

$$\frac{1}{\sqrt{\hat{g}_{1i}}} + \frac{1}{\sqrt{\hat{g}_{2j}}} = \frac{1}{\sqrt{\hat{g}_{12}}} + \frac{1}{\sqrt{\hat{g}_{21}}} \quad (18)$$

Equation (18) is similar to (13) of [5] based on DF relay system, thus we can prove that for the system with two subcarriers, the optimal SM is as $g_{1i} \sim g_{1j}$ and $g_{2i} \sim g_{2j}$ under Constraint A (see Appendix 1) in which the subcarrier pair with the lower gain is allocated to the lower power, by using the method in [5], which means that $g_{1i} \sim g_{1j}$ and $g_{2i} \sim g_{2j}$ can achieve more capacity than $g_{1i} \sim g_{2i}$ and $g_{2i} \sim g_{1j}$ with the constant total transmit power under Constraint A. When the total transmit power is constant, the objective of SM is equal to maximising energy efficiency with the given total power. Therefore the solution of this SM is a suboptimal of the original energy-efficiency problem without Constraint A. To maximise the capacity under a given total power, the probability of that power allocation satisfies Constraint A is large, that is, water-filling power allocation; hence the solution of the proposed SM may be an approximate optimal one.

In term of system with many subcarriers, Proposition 2 gives the optimal SM under Constraint A.
Virtual direct link can be represented as
\[ \tilde{g}_{ii} \sim \tilde{g}_i \]  \hspace{1cm} (19)

Proof: It is same as Proposition 3 in [5].

Therefore the optimal mapping in this paper is to match the subcarriers in the order of \( \tilde{G}_c \) and \( \tilde{G}_v \). The virtual direct link gain of each subcarrier pair is
\[ \tilde{g}_{ij} = \frac{\tilde{d}_{ij}}{d_{ij} + 1} \tilde{g}_{ij}^{\prime} \tilde{g}_{ij}^{\prime\prime} + \tilde{g}_i^{\prime \prime} \]  \hspace{1cm} (20)

where \( \tilde{d}_{ij} = (\tilde{g}_{ij}^{\prime \prime} / \tilde{g}_i^{\prime \prime}) \). According to Proposition 1, the virtual direct transmission channel gain is independent of total power \( \tilde{p}_{ij} \) on subcarrier pair, is dependent of the channel gains of each hop. If the virtual channel gain is based on the optimal SM, converting the relay link to the virtual direct link guarantees the optimality of original problem. Owing to the suboptimal SM, (20) is a suboptimal solution.

Following the rate of SM, the channel gains can be converted to rate of virtual direct link. The transmit power vectors on all subcarrier pairs of virtual direct link can be denoted as \( P = [\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_K] \), where \( \tilde{p}_i \in R_+ \). Then (5) can be rewritten as
\[ R(\tilde{P}) = \sum_{i=1}^K W \frac{1}{2} \log_2 (1 + \tilde{p}_i \tilde{g}_{ii}) \]  \hspace{1cm} (21)

Then the energy-efficiency problem of relay link is simplified as the one of virtual direct link. \( R(\tilde{P}) \) is strictly concave and monotonically increasing on \( \tilde{P} \). The energy efficiency with virtual direct link can be represented as
\[ (\tilde{P}^*) = \arg \max_\tilde{P} U(\tilde{P}) = \arg \max_\tilde{P} \frac{R(\tilde{P})}{\tilde{P}_c + \tilde{P}_T} \]  \hspace{1cm} (22)

where \( \tilde{P}_T = (1/s) \sum_{i=1}^K (\tilde{p}_{ii}) \).

3.2 Energy-efficient power allocation of virtual direct link

After SM, the two-hop link is converted to a virtual direct link. Thus, the method of the single-hop link can be used for power allocation. Based on this virtual direct link, it can be proved that \( U(\tilde{P}) \) has following properties.

Proposition 3: \( U(\tilde{P}) \) is strictly quasi-concave, if \( R(\tilde{P}) \) is strictly concave on \( \tilde{P} \). The unique global maximum of \( U(\tilde{P}) \) exists at \( \tilde{P}^* \).

Proof:
1. Quasi-concavity: According to [19], for any real number \( \alpha \), \( U(\tilde{P}) \) is strictly quasi-concave if and only if the upper contour set \( S_\alpha \) of \( U(\tilde{P}) \) is strictly convex, which can be denoted as
\[ S_\alpha = \{ \tilde{P} \in R^*_+ | U(\tilde{P}) \geq \alpha \} \]  \hspace{1cm} (23)

When \( \alpha \leq 0 \), \( S_\alpha = \{ \tilde{P} \in R^*_+ \} \) is strictly convex. When \( \alpha > 0 \), \( S_\alpha = \{ \tilde{P} \in R^*_+ | a\tilde{P}_c + a\tilde{P}_T - R(\tilde{P}) \leq 0 \} \). \( R(\tilde{P}) \) is strictly concave on \( \tilde{P} \), then \( -R(\tilde{P}) \) is strictly convex. Thus \( S_\alpha \) is also strictly convex, meanwhile the strict quasi-concavity of \( U(\tilde{P}) \) can be proofed. A local maximum of strictly quasi-concave functions is also a global maximum according to [20].

2. Uniqueness of global maximum: The partial derivative of \( U(\tilde{P}) \) with \( \tilde{p}_i \) is
\[ \frac{\partial U(\tilde{P})}{\partial \tilde{p}_{ij}} = R'(\tilde{P})(P_c + \tilde{P}_T) - R(\tilde{P}) \]  \hspace{1cm} (24)

If the optimal power \( \tilde{p}_{ij} \) exists such that \( (\partial U(\tilde{P})/\partial \tilde{p}_{ij}) = 0 \), it is unique, that is, if \( \tilde{p}_{ij} \) exists such that \( x(\tilde{p}_{ij}) = R(\tilde{P})(P_c + \tilde{P}_T) - R(\tilde{P}) = 0 \). We investigate existence of \( x(\tilde{p}_{ij}) \). The derivative of \( x(\tilde{p}_{ij}) \) is
\[ \frac{\partial x(\tilde{p}_{ij})}{\partial \tilde{p}_{ij}} = R''(\tilde{P})(P_c + \tilde{P}_T) \]  \hspace{1cm} (25)

\( R(\tilde{P}) \) is strictly concave on \( \tilde{P} \) and \( R''(\tilde{P}) < 0 \). Hence \( (\partial x(\tilde{p}_{ij})/\partial \tilde{p}_{ij}) < 0 \). Furthermore, \( x(\tilde{p}_{ij}) \) is strictly decreasing. Using the L’Hospital rule, we can obtain that
\[ \lim_{\tilde{p}_{ij} \to 0} x(\tilde{p}_{ij}) = \lim_{\tilde{p}_{ij} \to \infty} (R(\tilde{P})(P_c + \tilde{P}_T) - R(\tilde{P})) \]  \hspace{1cm} (26)

If \( \tilde{p}_{ij} \to 0 \) and \( x(\tilde{p}_{ij}) > 0 \), \( \tilde{p}_{ij} \) exists and \( U(\tilde{P}) \) is first strictly increasing and then strictly decreasing on \( \tilde{p}_{ij} \). If \( \lim_{\tilde{p}_{ij} \to 0} x(\tilde{p}_{ij}) \leq 0 \), \( \tilde{p}_{ij} = 0 \), and \( U(\tilde{P}) \) is strictly decreasing on \( \tilde{p}_{ij} \). Therefore the unique global optimal power vector \( \tilde{P}^* = [\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_K] \) for (21) exists.

According to Proposition 3, the global optimal power vector of virtual link exists and is unique. Unfortunately, this energy-efficient power allocation is non-linear problem. It is difficult to directly solve the optimal power vector according to (21). However, we can use the low complexity iterative algorithms binary search assisted ascent (BSAA) from [8] to search the optimal power vector. The global optimality of the optimal \( \tilde{P} \) is guaranteed by the strict quasi-concavity of \( U(\tilde{P}) \).

For finding the optimal \( \tilde{P} \), the gradient ascent method is used to produce a maximising sequence \( \tilde{P}^{(i)} \), \( i = 0, 1, 2, \ldots \), then
\[ \tilde{P}^{(i+1)} = \left[ \tilde{P}^{(i)} + \alpha \nabla U(\tilde{P}^{(i)}) \right]^+ \]  \hspace{1cm} (26)
4 Case with constraint

In the previous section, the power allocation is considered without any constraints. Therefore, in this section, we study the energy-efficient power allocation with one constraint, the minimum rate requirement or the maximum transmit power constraints of SN and RN.

4.1 Under the minimum rate requirement constraint

The required minimum rate is assumed $R_r$ bps. The energy-efficiency problem can be rewritten as

$$
\hat{\mathbf{P}}_r = \arg \max_p R(\hat{\mathbf{P}}) = \arg \max_p \frac{R(\hat{\mathbf{P}})}{P_c + P_T}
$$

s.t. $R(\hat{\mathbf{P}}) \geq R_r \quad (27a)$

If the optimal transmit power vector $\hat{\mathbf{P}}_r^*$ without constraint of (22) can satisfy the constraint (27a), $\hat{\mathbf{P}}_r^*$ is also the solution to (27), that is, $\hat{\mathbf{P}}_r^* = \hat{\mathbf{P}}_r^*$. Otherwise, (27) is equivalent to

$$
\hat{\mathbf{P}}_r = \arg \max_p \frac{R(\hat{\mathbf{P}})}{P_c + P_T} = \arg \min_p \sum_{i=1}^K \hat{p}_{i,i}
$$

s.t. $R(\hat{\mathbf{P}}) = R_r \quad (28a)$

Using the Lagrange multipliers, the optimisation of (28) can be rewritten as

$$
\min \mathcal{L}(\hat{\mathbf{P}}, \lambda) = \sum_{i=1}^K \hat{p}_{i,i} - \lambda_b (R(\hat{\mathbf{P}}) - R_r) \quad (29)
$$

Differentiating $\mathcal{L}(\hat{\mathbf{P}}, \lambda)$ with respect to $\hat{p}_{i,i}$, and setting result to zero, we have

$$
\frac{\partial \mathcal{L}(\hat{\mathbf{P}}, \lambda)}{\partial \hat{p}_{i,i}} = 1 - \lambda_b \frac{\partial R(\hat{\mathbf{P}})}{\partial \hat{p}_{i,i}} = 0 \quad \text{for all } i \quad (30)
$$

The $K+1$ simultaneous equations, consisting of (30) and the original constraint given in (28a), define the optimum solutions $(\hat{\mathbf{P}}_r, \lambda)$. Therefore, the use of a numerical method is suggested, such as the vector form Newton’s method [21], for evaluating $\hat{\mathbf{P}}_r$. Owing to the uniqueness of $\hat{\mathbf{P}}_r$, such method can yield a reasonable accurate $\hat{\mathbf{P}}_r$ in only a few iterations.

4.2 Under the maximum transmit power constraint

The maximum transmit power constraints of SN and RN are assumed as $P_S$ watt and $P_R$ watt, respectively. The energy-efficiency problem can be rewritten as

$$
\hat{\mathbf{P}} = \arg \max_{\mathbf{P}} U(\mathbf{P}) = \arg \max_p \frac{R(\mathbf{P})}{P_c + P_T}
$$

s.t. $\sum_{i=1}^K p_{i,i} \leq P_S \quad (31a)$

$$
\sum_{i=1}^K p_{i,i} \leq P_R \quad (31b)
$$

According to (16), if $P_S > \sqrt{\left(\frac{g_r}{g_F}\right) P_R}$, where $g_r$, $g_F$ are respectively denoted average channel gains on all the subcarriers of first and second hops, the constraints given in constraints (31a), (31b) are equivalent to

$$
\sum_{i=1}^K \hat{p}_{i,i} \leq \left(1 + \sqrt{\frac{g_F}{g_r}}\right) P_R
$$

If $P_S \leq \sqrt{\left(\frac{g_r}{g_F}\right) P_R}$, the constraints given in constraints
(31a), (31b) are equivalent to
\[
\sum_{i=1}^{K} \hat{p}_{ij} \leq \left(1 + \sqrt{\frac{g}{g'}}\right)P_S \quad (33)
\]
If the optimal transmit power vector \( \hat{P}^* \) without constraint of (22) can satisfy the constraint (32) or (33), \( \hat{P}^* \) is also the solution to (31), that is, \( \hat{P}^* = \hat{P}^s \). Otherwise, (31) can be converted to
\[
(\hat{P}^s) = \arg \max \limits_{\hat{P}} U(\hat{P}) = \arg \max \limits_{\hat{P}} R(\hat{P}) \quad (34)
\]
\[
\text{s.t.} \quad \sum_{i=1}^{K} \hat{p}_{ij} = P_{\max}
\]
\[
= \begin{cases} 
\left(1 + \sqrt{\frac{g}{g'}}\right)P_R, & P_S > \sqrt{\frac{g}{g'}}P_R \\
\left(1 + \sqrt{\frac{g}{g'}}\right)P_S, & P_S \leq \sqrt{\frac{g}{g'}}P_R 
\end{cases} \quad (34a)
\]
Using the method of Lagrange multipliers, the optimisation of (28) can be rewritten as
\[
L(\hat{P}, \lambda) = R(\hat{P}) - \lambda_p \left(\sum_{i=1}^{K} \hat{p}_{ij} - P_{\max}\right) \quad (35)
\]
Differentiating \( L(\hat{P}, \lambda_p) \) with respect to \( \hat{p}_i \) and setting result to zero, we have
\[
\frac{\partial L(\hat{P}, \lambda_p)}{\partial \hat{p}_{ij}} = \frac{\partial R(\hat{P})}{\partial \hat{p}_{ij}} - \lambda_p = 0 \quad \text{for all} \quad i \quad (36)
\]
Similarly, the vector supporting Newton’s method is suggested for evaluating \( \hat{P}^s \) by using (36) and the original constraint given in (34a).

In this paper, the sorting algorithm of subcarriers in each hop is heap sort, and the algorithm complexity of which is \( O(K \times \log_2 K) \). Thus the algorithm complexity of SM is \( O(K \times \log_2 K + K) \). If the optimal transmit power allocation of virtual direct link without constraint of (19) can satisfy the constraints, the algorithm complexity of power allocation is \( O(X) \), where \( X \) is the number of iterations in the iterative algorithms BSAA and is usually very small. Otherwise, the algorithm complexity of power allocation of virtual direct link is \( O(K \times Y) \), where \( Y \) is iterations during the vector-form Newton’s method. Therefore the total algorithm complexity of the proposed algorithm is \( O(K \times \log_2 K + K + X) \) or \( O(K \times \log_2 K + K + X) \).

5 Simulation results

In this section, we present selected results obtained by computer simulations to illustrate system performance using proposed power allocation method. The impacts of the proposed SM algorithm, the power allocation of virtual direct link, the virtual direct link and constraints on energy efficiency are analysed. The simulation parameters are listed in Table 1. All simulation results are averaged over many independent channel realisations.

In order to compare EEO with MAO and RAO and evaluate the algorithms in Sections 3 and 4 with the idea of virtual direct link, we simulate and verify the following five schemes:

1. **EEO w/o constraint**: the proposed energy efficient algorithm in Section 3 is used to allocate transmit power of each hop without any constraints.
2. **EEO w R:R**: the proposed energy efficient algorithm in Section 3 or 4.1 is used to allocate transmit power of each hop, subjected to the minimum rate requirement.
3. **EEO w (P_S and P_R)**: the proposed energy-efficient algorithm in Section 3 or 4.2 is used to allocate transmit power of each hop, subjected to the maximum transmit power constraint.
4. **MAO**: MAO based on proposed algorithm in Section 4.1 is used to allocate transmit power.
5. **RAO**: RAO based on proposed algorithm in Section 4.2 is used to allocate transmit power.

Figs. 3 and 4 demonstrate the influence of constraints on energy efficiency and the energy-efficient power allocation performance compared with the ones based on RAO and MAO. It is assumed that the distance between SN and RS is 750 m and the one between RN and DN is 750 m. The effect of the minimum rate requirement is presented in Fig. 3. In Fig. 3a, since EEO without constraint is not limited by any constraint, the change of minimum rate requirement has no effect on the result of power allocation. The energy efficiency of EEO without constraint is constant with the growth of rate requirement, whereas the energy efficiency of MAO is firstly increasing and then decreasing. The energy efficiency of MAO reaches a peak at nearly 6.6 Mbit/J, an equal value to the optimal value of EEO without constraint. It is also shown that the first half part of curve from EEO w R equals to the one from EEO without constraint, but the second half part of the curve is same as MAO. The reason is that if the rate requirement is small enough, the optimal solution of EEO without constraint is able to satisfy the rate constraint, which means that the solution of EEO w R equals to the one of EEO without constraint. Otherwise, the optimal solution cannot satisfy the constraint and the solution of EEO w R equals to the one of MAO. Figs. 3b and c illustrate the throughput and transmit power of the three schemes. From the figures, we can see that throughput and transmit power of MAO increase with the increase of minimum rate requirement. However, when the throughput and

<table>
<thead>
<tr>
<th>Table 1 Simulation parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>carrier frequency, GHz</td>
</tr>
<tr>
<td>system bandwidth, MHz</td>
</tr>
<tr>
<td>SN–RN, RN–DN lognormal shadowing, dB</td>
</tr>
<tr>
<td>number of points in full FFT</td>
</tr>
<tr>
<td># of used subcarriers</td>
</tr>
<tr>
<td># of the multipath</td>
</tr>
<tr>
<td>RN–DN SN–RN path loss</td>
</tr>
<tr>
<td>height of SN and RN, m</td>
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<tr>
<td>height of DN, m</td>
</tr>
<tr>
<td>thermal noise density, dBm Hz⁻¹</td>
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<tr>
<td>amplifier efficiency</td>
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<tr>
<td>total circuit power, W</td>
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power consumption of MAO are smaller than the ones of EEO without constraint, the energy efficiency of MAO increases with the throughput and power increased. When the throughput and power consumption of MAO are larger than the ones of EEO without constraint, the energy efficiency of MAO decreases with the increase of the throughput and power increased.

Fig. 4 demonstrates the influence of power limitation on energy efficiency. As shown in Fig. 4a, in relay link, the power limitations of SN and RN both have an effect on the energy efficiency. When the power limitation of RN is very small and equals to 0.01 W, the optimal solution of EEO without constraint cannot satisfy the constraint, consequently, the curves of EEO w (P_S and P_R) and RAO are coincident. If power limitation of SN is larger than a certain value, the total transmit power is only correlated with power limitation of RN, which can be explained by (16) and (32). In this case, the values of curves keep constant. When the power limitation of RN increases to 0.06 W, and the power limitation of SN is larger than a certain value, the optimal solution of EEO without constraint can satisfy the constraint. Therefore the second half part of the curves in EEO w (P_S and P_R) is coincident with in EEO without constraint. However, if the power limitation of SN is small enough, P_max is only related to the power limitation of SN. Otherwise, P_max is only related to the one of RNs, which can be explained by (34a). That is the reason why the second half part of curve from RAO stays unchanged when the power limitation of RN is 0.06 W. When the power limitation of RN increases to 0.1 W, providing enough loose limitation of RN, P_max is only correlated with the limitation of SN, explaining why the second half part of the curve from RAO does not keep constant in this case. The influences of power limitation on throughput and total power consumption in Figs. 4b and c are same as those in Figs. 3b and c, so the similar conclusions as Figs. 3b and c can be obtained from Figs. 4b and c.

Figs. 5 and 6 demonstrate the influence of constraint on the optimal energy-efficient relay position. Owing to the same path loss model used in SN–RN link and RN–DN link, the shadow fading are close to 0 dB after averaging the calculation results for many times. Therefore, without considering the constraint, the optimal energy-efficient relay position obtained from simulation is near the middle value between SN and DN. It can be seen in Fig. 5 that the minimum rate requirement only has an effect on the value of energy efficiency. The rate requirement does not affect the optimal energy-efficient relay position. No matter how much rate requirement is, the optimal energy-efficient relay position under rate requirement is same as that without rate requirement, but the less the rate requirement is, the greater

**Fig. 3** Influence of minimum rate requirement on energy efficiency, throughput and total power consumption 156 × 105 mm

* a The minimum rate requirement, R_r bps
* b The minimum rate requirement, R_r bps
* c The minimum rate requirement, R_r bps
the energy efficiency is. Although it comes to power limitation, as seen from Fig. 6, the power limitation influences not only the value of the energy efficiency but also the optimal energy-efficient relay position. When the power limitation of RN is constant, the optimal relay position moves to the right and the energy efficiency goes up with the increase of the power limitation of SN. Although the power limitation of SN is constant, the

![Image](image_url)
optimal relay position moves to the left and the energy-efficiency increases with the raising of the power limitation of RN.

Furthermore, we analyse the influence of SM on energy efficiency. The distance between SN and RN is assumed as 750 m. In Fig. 7a, the SM algorithm in Section 3 is only compared with the case without SM. The number of subcarriers in each hop is assumed 128. In Fig. 7b, the proposed SM algorithm in Section 3 is compared with the exhaustive search (ES) algorithm and the case without SM (without SM), that is, $\tilde{g}_i$. The number of subcarriers in each hop is assumed five because of the algorithm complexity of ES algorithm. The curve of EEO w SM nearly coincides with the curve of EEO w ES in Fig. 7b, proving that the results of proposed SM method in Section 3 is an approximate optimal solution. As shown in Fig. 7, the total energy efficiency decreases with the increasing distance between RN and DN. The differences between EEO w SM and EEO without SM also grow gradually. The reason is that the farther the distance is, the greater the differences among subcarriers are, and the more the gain is obtained from SM. In addition, the more the number of subcarriers is, the greater the differences between subcarriers are. Hence, the difference between EEO w SM and EEO without SM in Fig. 7a is bigger than in Fig. 7b. Since only one user is considered in this paper, the difference between the subcarriers in each hop is only caused by small scale Rayleigh fading. The difference of fading between subcarriers is not so significant. Hence SM in single user system cannot obtain much gain. Especially, when the user is near the RN, SM is trivial. If the user locates at the coverage edge of the relay, the gain of energy efficiency from SM can achieve 6.15% in Fig. 7a.

Fig. 7 Influence of the proposed SM algorithm on energy efficiency 180 × 139 mm

a Distance between RN and DN, m
b Distance between RN and DN, m

Fig. 8 Influence of virtual direct link on energy efficiency 147 × 107 mm
Finally, the effect of virtual direct link on energy efficiency is analysed in Fig. 8. The proposed power allocation based on virtual direct link and SM is compared with the algorithm from [2], which uses Hungarian algorithm and Lagrangian dual method for RA optimisation but does not convert two-hop relay link to a virtual direct link. From Fig. 8, the obtained energy efficiency of RA optimisation is very close to and a little lower than the algorithm from [2]. The gap between RAO and the algorithm from [2] is caused by the suboptimal SM, because the proposed power allocation of virtual direct link is optimal. However, the algorithm complexity of the algorithm from [2] is $O(K^3 \times Z)$, where $Z$ is the iteration of the optimal algorithm in [2]. Hence, virtual direct link declines energy efficiency a little, but the algorithm complexity of proposed algorithm decreases greatly because of the introduction of virtual direct link.

6 Conclusion

This paper proposes an appropriate power allocation scheme to maximise the energy efficiency in OFDM-based AF relay link. By taking the characteristic of AF relay, a SM method is proposed. Then the AF relay link is converted to a virtual direct link to help decrease the algorithm complexity of power allocation. The unique optimal transmit power vector is proved to be existed and final obtained. Last but not least, the cases with constraints are also analysed. Simulation results show that the proposed algorithm can improve energy efficiency, compared with the traditional RAO and MAO methods. Nevertheless, the more rigorous the constraints are, the smaller the energy efficiency is. For optimal energy-efficient relay position, the power limitations of SN and RN both affect the result, whereas rate requirement does not influence it. From the simulation, it can be known that the proposed SM method approximates optimal value and the suboptimal solution of the proposed two-step algorithm is very close to the optimal. It can also be known that the greater difference between subcarriers is, the more gain of SM can be achieved. Hence in single user system, the gain from SM is not so large. Although different from single user system, SM may obtain more gain in multiuser system. We will prove it in the next work for the same problem in multiuser system.

7 Acknowledgments

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8 References


9 Appendix

9.1 Derivation of Constraint A

For the two-ways of SM, $\tilde{g}_{1} \sim \tilde{g}_{1}, \tilde{g}_{2} \sim \tilde{g}_{2}$ and $\tilde{g}_{1} \sim \tilde{g}_{2}, \tilde{g}_{2} \sim \tilde{g}_{1}$, based on (18), the virtual direct channel gains satisfy the following constraint

$$\frac{1}{\sqrt{\tilde{g}_{1,1}}} + \frac{1}{\sqrt{\tilde{g}_{1,2}}} = \frac{1}{\sqrt{\tilde{g}_{1,2}}} + \frac{1}{\sqrt{\tilde{g}_{2,1}}} = H$$

where the parameter $H$ is a constant. For the first way, it is assumed that

$$\frac{1}{\sqrt{\tilde{g}_{1,1}}} - \frac{1}{\sqrt{\tilde{g}_{2,1}}} = x_{1}, \quad (H \geq x_{1} \geq 0)$$

For the second way, without loss of generality, we can obtain

$$\frac{1}{\sqrt{\tilde{g}_{2,2}}} - \frac{1}{\sqrt{\tilde{g}_{2,1}}} = x_{2}, \quad (H \geq x_{1} \geq 0)$$
Therefore it can be obtained that

\[
\sqrt{\hat{g}_{1,1}} = \frac{2}{H + x_1}, \quad \sqrt{\hat{g}_{2,2}} = \frac{2}{H - x_1},
\]

\[
\sqrt{\hat{g}_{1,2}} = \frac{2}{H + x_2}, \quad \sqrt{\hat{g}_{2,1}} = \frac{2}{H - x_2},
\]

and

\[\sqrt{\hat{g}_{1,1}} \leq \sqrt{\hat{g}_{2,2}}, \quad \sqrt{\hat{g}_{1,2}} \leq \sqrt{\hat{g}_{2,1}}\]

For the kth SM way, the corresponding total channel capacity is

\[
R_k(P_1, P_2) = \frac{W}{2} \log_2 \left( 1 + \frac{P_1}{(H + x_k)^2} \right) + \frac{W}{2} \log_2 \left( 1 + \frac{P_1}{(H - x_k)^2} \right)
\]

where \(P_1, P_2 \geq 0\) are the allocated power of the two subcarrier-pairs, respectively. Then the partial derivative of the channel capacity with respect to \(x_k\) can be obtained as (see equation at the bottom of the page)

If we want \((\partial R_k(P_1, P_2)/(\partial x_k)) > 0\), it should satisfy the constraint \(P_2 - P_1 \geq 0\), which means that subcarrier pair with the lower gain is allocated to the lower power. This constraint is denoted as ‘Constraint A’. Then under Constraint A, the total channel capacity is a monotonically increasing function of \(x_k\). This means that, under Constraint A, the larger the difference between the equivalent channel gains are the larger the total channel capacity is. Meanwhile, it is clear that the difference between the virtual direct channel gains of the first way is larger than the one of the second way. Therefore, for the same power allocation, the relationship of the total channel capacities of the two ways can be expressed: \(R_1(P_1, P_2) \geq R_2(P_1, P_2)\).